ICCG-16 ABSTRACT

Linear stability of binary alloy solidification for unsteady growth rates

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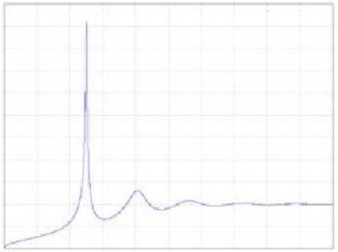
Abstract

An extension of the Mullins and Sekerka (MS) linear stability analysis to the unsteady growth rate case is considered for dilute binary alloys. In particular, the stability of the planar interface during the initial solidification transient is studied in detail numerically. The rapid solidification case, when the system is traversing through the unstable region defined by the MS criterion, has also been treated. It has been observed that the onset of instability is quite accurately defined by the "quasi-stationary MS criterion", when the growth rate and other process parameters are taken as constants at a particular time of the growth process. A singular behavior of the governing equations for the perturbed quantities at the constitutional supercooling demarcation line has been observed. However, when the solidification process, during its transient, crosses this demarcation line, a planar interface is stable according to the linear analysis performed.

LINEAR STABILITY OF BINARY ALLOY SOLIDIFICATION FOR UNSTEADY GROWTH RATES



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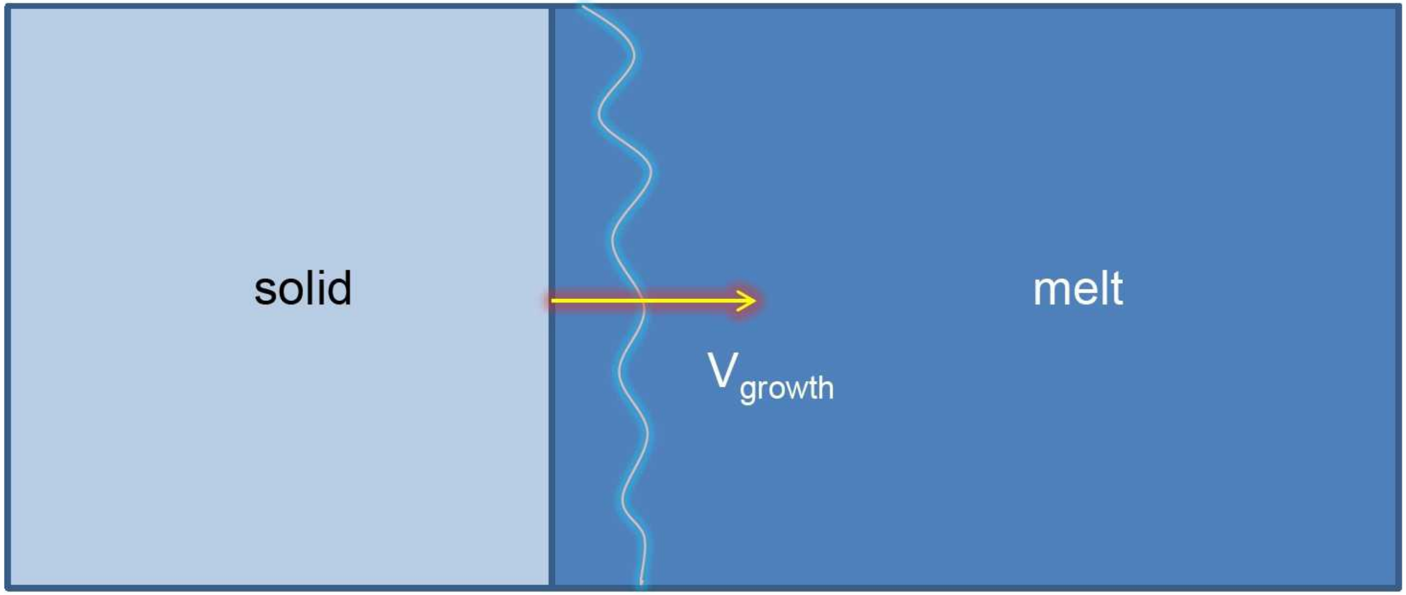
Stability of Planar Accelerated Interfaces



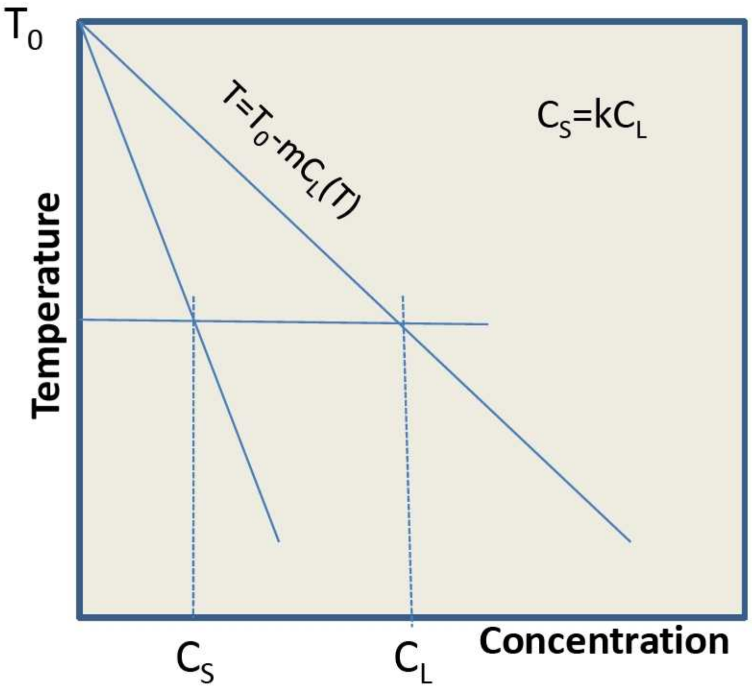
- Mullin-Sekerka interface stability – only for stationary motion
- Warren and Langer (Phys.Rev E, vol 47, 1993) - accelerated interfaces

Warren and Langer ignored accelerated terms in boundary conditions.

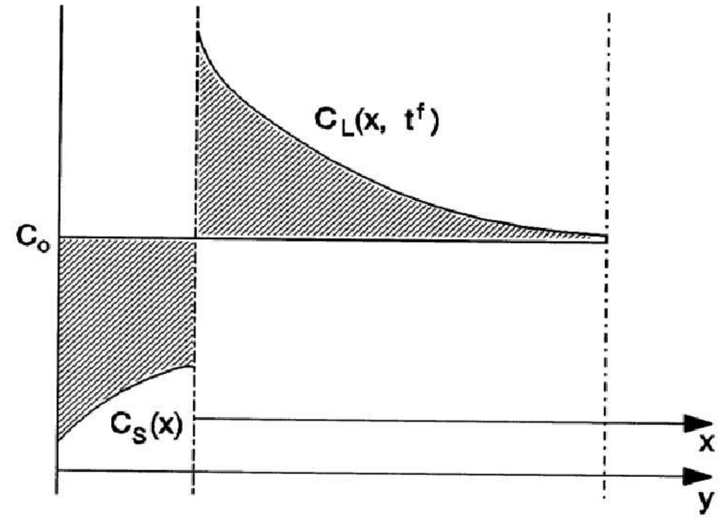
Question – how this approximation affects linear stability

Directional Solidification of Binary Alloy

- Diffusion controlled growth
- Frozen temperature approximation $T(z)=T_0+G(z-V_0t)$
- Local equilibrium at interface
- One dimensional case - planar interface growth



G – thermal gradient in liquid phase
V_0 – furnace translation rate



Model for Accelerated Growth of Planar Interface

Solute diffusion in liquid phase in the coordinate system co-moving with the interface

$$\frac{\partial c}{\partial t} - V\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial z^2}$$

Interface boundary condition

$$D\frac{\partial c}{\partial z} = \left(c_S - c_L\right)V = -c(1-k)V$$

Interface velocity

$$V = V_0 + \frac{m}{G}\frac{\partial c}{\partial t}$$

No latent heat effects

Initial Conditions

At time t=0:

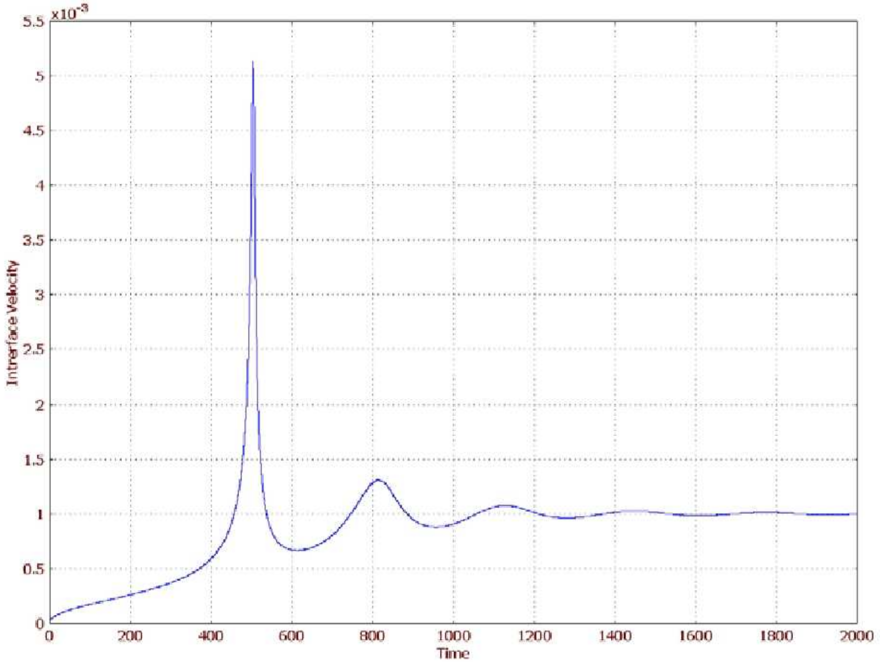
Interface velocity is zero
Solute concentration in liquid phase is C_0 everywhere
Solute concentration in solid is kC_0
Temperature at the interface position z_0 is T_0-Gz_0

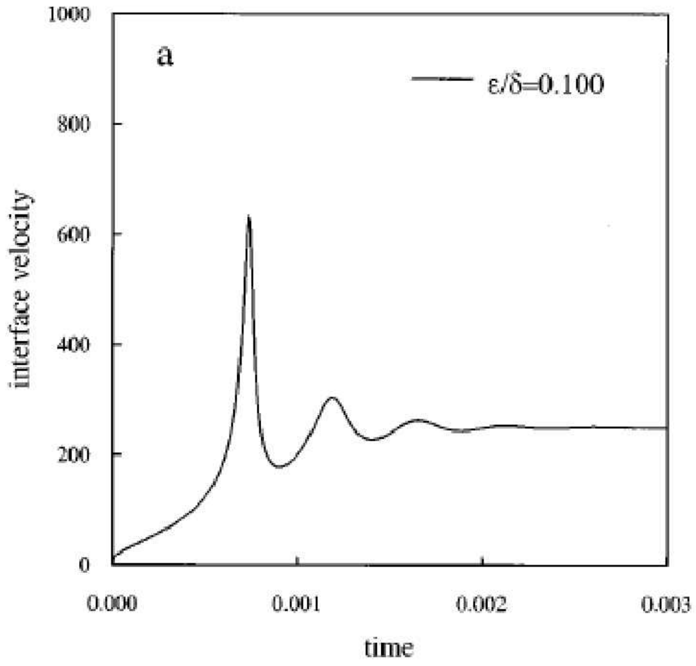
Initial transient during directional solidification was treated by Tiller in 1953. Some recent works include:

- W. Huang et al, J.Cryst.Growth 182 (1997) 212-218.
- D. Ma et al, J. Cryst.Growth 169 (1996) 170-174.
- A. Karma, A. Sarkissian, Phys.Rev. E 47,(1993) 513-533.
- B. Caroli et al, J.Cryst. Growth 132 (1993) 377-388.
- Majchrzak et al, J. Mater. Proc. Techn. 78 (1998) 122-127.
- Ch. Charach et al, Phys. Rev. E 54 (1996) 588-598.
- M.Conti, Phys.Rev. E 60 (1999) 1913-1920.

Numerics

Numerical solution by COMSOL 3.3





M. Conti, Phys Rev. E (1999)
Phase-field model

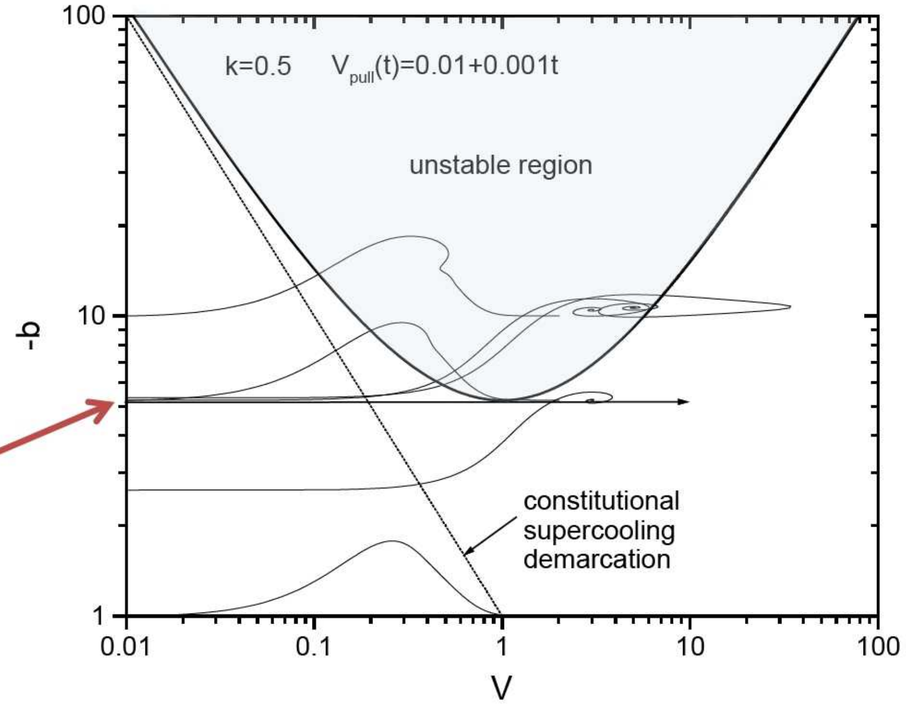
Mullin-Sekerka C-V Diagram

Scaling

$$V_{\min} = f(k) D \sqrt{\frac{G}{\Gamma}} \qquad f(k) = \sqrt{\frac{1-4k+\sqrt{1+8k}}{2k(1-k)}} \qquad \omega_{\min} = \sqrt{\frac{G}{\Gamma}}$$

$$b = \frac{m c_0 V}{GD}$$

$$b_{\min} = \frac{2k\left(2k-1+\sqrt{1+4f(k)^{-2}}\right)}{(k-1)\left(\sqrt{1+4f(k)^{-2}}-1\right)}$$



Governing Equations for Harmonically Perturbed Interfaces

Solute concentration in liquid phase in the coordinate system co-moving with the planar interface

$$c = c_0(z,t) + c_\omega(z,t)e^{i\omega x}$$

Linearized diffusion equation for the perturbed concentration

$$\frac{\partial c_\omega}{\partial t} - V\frac{\partial c_\omega}{\partial z} = \left(\frac{\partial^2 c_\omega}{\partial z^2} - \omega^2 c_\omega\right)$$

Interface boundary condition

$$\frac{\partial c_\omega}{\partial z} + c_\omega A(t) + B(t)\frac{\partial c_\omega}{\partial t} = 0$$

$$A(t) = \frac{\partial c_0}{\partial t}\frac{1}{M} + (1-k)V - \frac{VkG_c}{M} + \frac{(1-k)(1+c_0)\dot{G}_c}{M^2}$$

$$B(t) = \frac{(1-k)(1+c_0)}{M}$$

$$M = \frac{1+\omega^2\Gamma'}{b} - G_c$$

$$G_c = -(c_0+1)(1-k)V$$

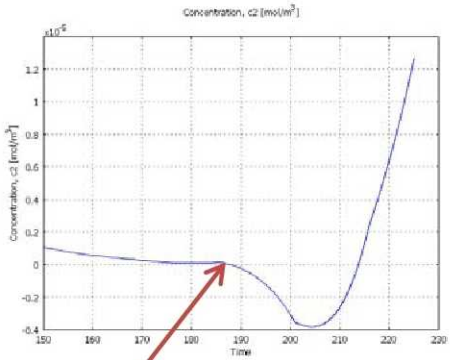
$$\Gamma' = \Gamma\frac{V_0^2}{GD^2}$$

Amplification of Perturbations

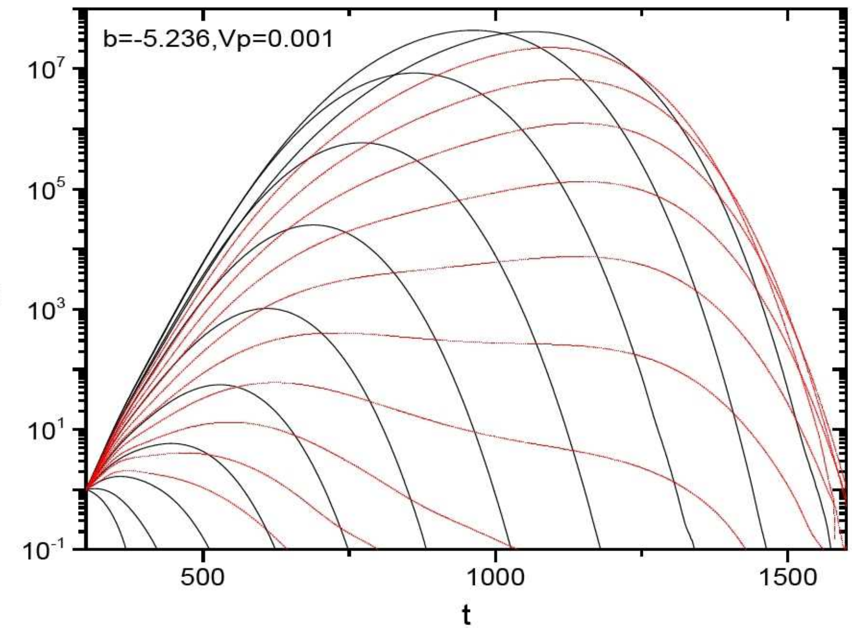
Traversing through the singular region for which M=0

$$M = \frac{1+\omega^2\Gamma'}{b} - G_c \qquad b_{sing} = \frac{1+\omega^2\Gamma'}{(k-1)(1+c_0)V}$$

C_0 is the concentration at the interface
For ω=0 and C_0 for the stationary case, this is the constitutional supercooling criterion



Singularity crossing



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Conclusions

- Linear stability of a planar solidification front has been studied for the case of accelerated growth rates. The approximate model proposed by Warren and Langer has been compared with accurate numerical modeling.
- Behavior near a singularity point in the boundary conditions for the perturbed quantities has been studied. No special effects has been observed.
- Stable accelerated interface are possible when no entering into the Mullin-Sekerka unstable zone occurs.